Worksheet 2-1: Different Forms of Quadratic Functions

Each quadratic function has three different ways to write its equation.

<table>
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<th>Form</th>
<th>Equation</th>
<th>Notes</th>
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<tbody>
<tr>
<td>Standard Form</td>
<td>( y = ax^2 + bx + c )</td>
<td>( y )-intercept can be easily seen from the equation</td>
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<tr>
<td></td>
<td>( y )-intercept = ( c )</td>
<td></td>
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<tr>
<td>Vertex Form</td>
<td>( y = a(x - h)^2 + k )</td>
<td>vertex can be easily seen from the equation</td>
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<td></td>
<td>vertex = ((h, k))</td>
<td></td>
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<tr>
<td>Intercept Form</td>
<td>( y = a(x - r)(x - s) )</td>
<td>( x )-intercepts can be easily seen from the equation</td>
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<tr>
<td></td>
<td>( x )-intercepts = ( r ) and ( s )</td>
<td>*This is also called the factored form</td>
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A function is:

- **positive** when \( f(x) > 0 \) for given values of \( x \)  
  (the part of the graph above the \( x \)-axis)
- **negative** when \( f(x) < 0 \) for given values of \( x \)  
  (the part of the graph below the \( x \)-axis)

1. Each function is in standard form.
   (1) State the direction of opening.
   (2) State whether the vertex is a maximum or minimum.
   (3) State the \( y \)-intercept.

   (a) \( f(x) = 2x^2 + x - 3 \)  
   (b) \( g(x) = -5x^2 + 10x + 15 \)
2. Each relation is in vertex form.

   (1) State the direction of opening, vertex, maximum or minimum value, axis of symmetry, and y-intercept.
   (2) Graph the function.
   (3) State the intervals for which the function is positive or negative.
   (4) State the intervals for which the function is increasing or decreasing.

   (a) \( f(x) = -2(x + 1)^2 - 3 \)
   (b) \( g(x) = \frac{1}{2}(x - 5)^2 - 8 \)
3. Each relation is in factored or intercept form.

   (1) Identify the direction of opening, $x$-intercepts, vertex, maximum or minimum value, axis of symmetry, and $y$-intercept.

   (2) Graph the function.

   (3) Identify the intervals for which the function is positive or negative.

   (4) Identify the intervals for which the function is increasing or decreasing.

(a) $f(x) = 2(x + 3)(x - 1)$

(b) $g(x) = -(x - 1)(x - 5)$
4. Determine the zeros of each quadratic relation.

(a) \( f(x) = 3(x + 8)(x - 3) \)
(b) \( y = -2(x - 3)(x - 5) \)
(c) \( g(x) = (x + 6)^2 \)

5. Which relation has more than one zero, only one zero, and no zero at all? Explain.

(a) \( f(x) = -5(x + 8)^2 \)
(b) \( y = 3(x - 7)^2 + 4 \)
(c) \( g(x) = -(x + 6)^2 + 5 \)

Answers: 4. (a) \(-8\) and \(3\), (b) \(3\) and \(5\), (c) \(-6\) (repeated zeros); 5. (a) one zero, (b) no zero, (c) two zeros